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A PROPOSED DIAGNOSTIC METHOD FOR CYLINDRICAL PLASMAS

on

J. SHMOYS

Research Report PIBMRI-828-60

for

AIR FORCE CAMBRIDGE RESEARCH CENTER
Contract No. AF-19(604)-4143

May 16, 1960

ELECTROPHYSICS GROUP

MRI

POLYTECHNIC INSTITUTE OF BROOKLYN
MICROWAVE RESEARCH INSTITUTE

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A PROPOSED DIAGNOSTIC METHOD FOR CYLINDRICAL PLASMAS

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Title Page
Acknowledgement
Abstract
Table of Contents
14 Pages of Text
Figures

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ABSTRACT

For a cylindrically symmetric plasma column whose electron density N is a slowly varying, monotonically decreasing function of radius r, it is possible to calculate explicitly both the diffraction pattern from the knowledge of N(r) and, conversely, N(r) from the knowledge of the diffraction pattern. If the diffraction pattern is obtained experimentally, N(r) can be calculated by a cumbersome numerical procedure. Instead of doing this, the diffraction pattern can be approximated by one of a family of convenient analytical expressions for which the integration can be carried out easily. Alternatively, one can attempt to infer N(r) by assuming a functional form for N(r) with one or more parameters, calculate the diffraction pattern and compare it with the observed one.

TABLE OF CONTENTS

1. Introduction

It is possible to determine the electron density distribution in a lossless plasma cylinder by illuminating it with a plane electromagnetic wave and observing the diffraction pattern. Let us assume that the plasma cylinder is long compared to wavelength and that the plasma density depends on the distance from the axis alone. This plasma cylinder is illuminated by a weak plane electromagnetic wave incident at right angles to the axis, polarized so that its electric vector is parallel to the axis. The electric field has a single axial component E_z which satisfies the wave equation

$$\left[\nabla^2 + k^2(r)\right] E_z = 0 \tag{1}$$

where

$$k^2(r) = \omega^2 \mu_o \epsilon_o \left(1 - \frac{N(r)e^2}{\epsilon_o m \omega^2}\right) \tag{2}$$

$$= k_o^2 - N(r)\mu_o e^2/m$$

and

ω - angular frequency of the wave

μ_o, ϵ_o - magnetic permeability and dielectric constant of free space

e, m - electronic charge, mass

$N(r)$ - electron density (no. of electrons/m^3)

k_o - $\omega\sqrt{\mu_o \epsilon_o}$, free space wavenumber

Far away from the plasma cylinder the field consists of the incident plane wave and an outgoing cylindrical wave. From the dependence of the power density in the cylindrical wave on the angular direction or on the frequency it is possible to deduce the radial distribution $N(r)$. This inverse scattering

problem has received considerable attention in quantum mechanics. The wave equation then is

$$\left[\nabla^2 + k^2(r)\right]\psi = 0 \tag{3}$$

where

$$k^2(r) = \frac{2m}{\hbar^2} E - \frac{2m}{\hbar^2} V(r)$$

The energy E here plays the same role as the frequency does in the case of plasma, and the potential energy $V(r)$ corresponds to the electron density distribution. In quantum mechanics one normally tries to construct the potential function $V(r)$ from known scattering data (either total scattering cross-section, or differential scattering cross-section in a fixed direction) as a function of energy. This is basically the same procedure as is used, for example, in ionospheric sounding, where frequency is varied. The method dealt with here utilizes scattering data in all directions at a single energy (frequency).

Whatever type of data is used, the exact inversion problem is an extremely difficult one to solve[1]. For this reason we may resort to an approximation. If the variation of properties of the medium is slow, i.e., the relative change per wavelength is small, then we can resort to a WKB or geometric-optical approximation. We shall assume that the geometric-optical approximation is valid, calculate the differential scattering cross-section from the electron density distribution on that basis, and then invert that relationship so as to obtain the electron density distribution from the differential scattering cross-section.

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1. I. M. Gelfand and B. M. Levitan, Izv. Akad. Nank SSSR, Mat. Ser. 15, 309 (1951)
Doklady Akad. Nank SSSR, 77, 557 (1951)

2. Differential Scattering Cross-section and Impact Parameter

Consider a plane wave incident perpendicularly on a plasma column; the rays are initially parallel straight lines (See Fig. 1a). When these rays enter the ionized region, they are deflected. The impact parameter b for a given ray is defined as the distance between the central ray (ray in the plane of symmetry of the plasma column) and the given ray. If the incident wave carries unit power per unit width (this is a two-dimensional geometry), then the power contained between rays with impact parameter b and $b + db$ is db. If these rays emerge from the plasma at angles θ and $\theta + d\theta$, then, on the basis of geometric-optical consideration,

$$\sigma d\theta = db \tag{4}$$

where σ is the scattered power per unit angle, or the differential scattering cross-section. We see that $\sigma(\theta)$ and $\theta(b)$ are simply related as follows:

$$\sigma(\theta) = \frac{db}{d\theta} . \tag{5}$$

The first step in the inversion process is to calculate $\theta(b)$, given $\sigma(\theta)$. Even if, as we shall assume, the ionization density N is a monotonic decreasing function of r, there may be a difficulty in this step. We have to distinguish between soft, or penetrable, obstacles, and hard, or impenetrable ones. If the plasma column is impenetrable then the central ray is deflected through 180^{o}, while distant rays are not deflected (see Fig. 1b). In that case, $b(\theta)$ is a single-valued function, and there is no difficulty in integrating $\sigma(\theta)$ to obtain $b(\theta)$, and then to invert that relation to obtain $\theta(b)$. If, on the other hand, the plasma column is penetrable (Fig. 1c), the central ray is not deflected, and other rays are deflected through various angles ranging from zero to a certain maximum value θ_o (maximum diffraction

angle). Hence there are two rays deflected in any given direction θ, and $b(\theta)$ is double-valued.

In the latter case the differential scattering cross-section will oscillate rapidly as a function of θ as a result of interference between the two rays deflected in the same direction. At the peaks the two intensities add, at the minima they subtract. If we plot intensity (square root of differential cross-section) vs. θ, we can readily construct the curves showing the variation of the sum, I^+, and the difference, I^-, of the two ray intensities I_1 and I_2 (see Fig. 2). Assuming $I_2 > I_1$, we have

$$\begin{aligned} \sqrt{\sigma_1} &= I_1 = \frac{1}{2}(I^+ - I^-) \\ \sqrt{\sigma_2} &= I_2 = \frac{1}{2}(I^+ + I^-) \end{aligned} \tag{6}$$

There are difficulties to be expected in reconciling experimental data with ray theory. If we start from a dependence of the impact parameter b on θ, such as shown on Fig. 2c then we notice that the intensity associated with the inner ray (1) becomes infinite at θ_o, and the intensity associated with the outer ray (2) becomes infinite at $\theta = 0$ and at θ_o. No such singularities will, of course, be observed. They appear in the geometric-optical approximation because the approximation is not valid at those values of θ as a consequence of the behavior of the caustic surface in this case. Should the ionization density be identically zero outside some finite radius then the singularity at $\theta = 0$ may disappear. In any case, we can utilize only those portions of the curves which are not too close to $\theta = 0$ or $\theta = \theta_o$. The problem is then to construct all of the curve on Fig. 2c from the knowledge of the slope of this curve (or, rather, the slopes of its two branches) in the range $\alpha < \theta < \theta_o - \beta$.

It can be shown that, for small values of impact parameter, the ray deflection is proportional to the impact parameter. This means that we can extend the curve I_1 to $\theta = 0$ simply by assuming that I_1 remains constant for $0 < \theta < \alpha$. Thus we obtain b vs. θ from the origin to $\theta = \theta_o - \beta$. We shall assume that while the scattering data in the range $\theta_o - \beta < \theta < \theta_o$. cannot be used, the value of θ_o is known, and that the value impact parameter at $\theta = \theta_o - \beta$, as well as the slope of the b vs. θ curve (the slope is simply the differential scattering cross-section σ), are known, and finally that the shape of the θ vs. b curve is parabolic (see Fig. 2d), i.e.,

$$(\theta_o - \theta) \propto (b_o - b)^2$$

With these assumptions, the difference between the two values of impact parameter at $\theta = \theta_o - \beta$, $b_2 - b_1$ is given by

$$b_2 - b_1 = 4\beta\sigma(\theta_o - \beta) \tag{7}$$

where $\sigma(\theta_o - \beta)$ is the differential scattering cross-section in the direction $\theta = \theta_o - \beta$. Since b_1 is known, equation (7) establishes a point on the second branch of the b vs. θ curve. From that point we can continue the curve by using experimental values of scattering cross-section again.

3. Ray Path and Deflection Angle

The differential equation satisfied by the ray path $r(\theta)$ in a cylindrically stratified medium can be readily obtained from Snell's law. If the dielectric constant is a function of r, then Snell's law takes on the form

$$\sqrt{\epsilon(r)/\epsilon_o}\; r \sin \alpha = \text{constant} \tag{8}$$

where α is the angle between the ray direction and the radial direction

(see Fig. 3). Since for large r, $\epsilon \to \epsilon_o$, $r \sin \alpha \to b$, the constant on the right-hand side of (8) is simply the impact parameter b of the given ray. From triangle PP'Q (see Fig. 3) we see that

$$\left(\frac{r d\theta}{dr}\right)^2 = \tan^2 \alpha \ . \tag{9}$$

Eliminating α between equations (8) and (9), we obtain the differential equation of the ray

$$(r d\theta/dr)^2 = (r^2 \epsilon / b^2 \epsilon_o - 1)^{-1} \tag{10}$$

In the case of a lossless plasma the dielectric constant is

$$\epsilon/\epsilon_o = (k/k_o)^2 = 1 - N(r)e^2/\epsilon_o m\omega^2 \tag{11}$$

Let us introduce a notation which has the double purpose of making the relation (11), and those which follow, more compact, and at the same time bring out the similarity of the problem at hand and the corresponding one in particle mechanics; let us introduce

$$V(r) = N(r)e^2/\epsilon_o m\omega^2 \ . \tag{12}$$

Combining equations (10), (11) and (12) yields the final form of the differential equation of the ray with the impact parameter b

$$\left|\frac{d\theta}{dr}\right| = r^{-2}\left[b^{-2} - r^{-2} - b^{-2}V(r)\right]^{-1/2} \tag{13}$$

The angle between the direction along which the ray comes in from infinity, and the direction along which it goes out to infinity again is then

$$2\int_{r_o}^{\infty} \left|\frac{d\theta}{dr}\right| dr \ .$$

The deflection angle of the ray, i.e., the limiting value of the angular coordinate as $r \to \infty$, becomes

$$\theta(b) = \pi - 2 \int_{r_o}^{\infty} r^{-2} \left[b^{-2} - r^{-2} - b^{-2} V(r) \right]^{-1/2} dr \qquad (14)$$

where r_o, the distance of closest approach for this ray, is the zero of the expression in brackets ($\frac{dr}{d\theta} = 0$ at r_o).

We will now investigate the function $\theta(b)$ for various ionization density distributions. We have to distinguish between two types of distributions: penetrable and impenetrable. The illustrative examples will be of the former type. We shall consider three ionization distributions: (1) uniform (2) parabolic and (3) modified parabolic. The mathematical definitions of these three distributions, as well as the relations between the deflection angle and the corresponding impact parameter are listed in Table I. The functions $\theta(b)$ were obtained simply by evaluating the integral in Eq. (14) with the appropriate distribution function $V(r)$. The results listed in Table I are also shown graphically on Figs. 4, 5, and 6. The case of the uniform cylinder is different in character from the two others because the medium is discontinuous in this case. The geometric optical field consists of refracted, reflected and many multiply reflected and refracted contributions. If we consider only the refracted ray, then the deflection angle increases monotonically until $b = a/\sqrt{1 - V_o}$; for larger values of impact parameter the incident ray is totally reflected, until, for $b > a$, the incident ray completely misses the cylinder and is not deflected. The remaining two models present no such complications. In the case of a parabolic distribution the deflection angle is identically zero if the ray misses the cylinder, i.e., $b > a$, while in the modified parabolic distribution in which the extent of plasma is infinite, the deflection angle tends to zero as b increases.

TABLE I

$V(r)$	$\theta(b)$	θ_o	b_o
$V(r)=V_o,\ r<a$ $=0\ ,\ r>a$	$(b/a)^2 < 1 - V_o$ $\theta(b)=2\left(\sin^{-1}\dfrac{b/a}{\sqrt{1-V_o}} - \sin^{-1}\dfrac{b}{a}\right)$	$2\sin^{-1}\sqrt{V_o}$	$a\sqrt{1-V_o}$
$V(r)=V_o\left[1-\left(\frac{r}{a}\right)^2\right],\ r<a$ $=0\ ,\ r>a$	$\theta(b)=\sin^{-1}\dfrac{V_o\sqrt{\left(\frac{b}{a}\right)^2-\left(\frac{b}{a}\right)^4}}{\sqrt{V_o\left(\frac{b}{a}\right)^2+\left(\frac{1-V_o}{2}\right)^2}}$	$\sin^{-1}V_o$	$a\sqrt{\dfrac{1-V_o}{2}}$
$V(r)=V_o\left[1-\frac{1}{2}\left(\frac{r}{a}\right)^2\right],\ r<a$ $=\frac{1}{2}V_o\left(\frac{a}{r}\right)^2\ ,\ r>a$	$(b/a)^2 < 1 - \dfrac{V_o}{2}$: $\theta(b)=\dfrac{\pi}{2}+\sin^{-1}\dfrac{\left(\frac{b}{a}\right)^2-\frac{1-V_o}{2}}{\sqrt{\left(\frac{1-V_o}{2}\right)^2+\frac{V_o}{2}\left(\frac{b}{a}\right)^2}} - \dfrac{\frac{2b}{a}\sin^{-1}\sqrt{\left(\frac{b}{a}\right)^2+\frac{V_o}{2}}}{\sqrt{\left(\frac{b}{a}\right)^2+\frac{V_o}{2}}}$ $(b/a)^2 > 1 - \dfrac{V_o}{2}$: $\theta(b) = \pi\left(1-\dfrac{1}{\sqrt{1+\frac{V_o}{2}\left(\frac{a}{b}\right)^2}}\right)$		

It is interesting to observe the dependence of the maximum ray deflection angle on maximum density V_o for the two continuous models. The maximum deflection angle should be easily observable since outside this angle there is no geometric optical scattering, only true diffraction. At sufficiently high frequencies, the differential scattering cross-section should drop rapidly to very small values at this angle. We see from Fig. 7 that there is very little difference between the two models in their effect on the maximum deflection angle. Therefore we may conjecture that, so long as the variation of density near the center of the plasma column in quadratic, the maximum density can be inferred from the maximum deflection angle with reasonable accuracy. Since the relation between the two quantities is particularly simple in the case of the parabolic model, we can make use of it, thus concluding that

$$V_o \approx \sin\theta_o \tag{15}$$

may be a fair approximation for a great variety of smoothly varying distributions.

4. The Inverse Problem

We shall now proceed to invert equation (14), i.e., to solve it for $V(r)$ if $\theta(b)$ is given. In order to accomplish this, we shall reduce the equation to one of Abel type, and then use the well-known solution of the Abel integral equation. The procedure used here is identical with that employed in a previous solution of the integral equation (14).[2] It is convenient to introduce the new variables x, u, v, and w, defined as follows:

2. J. B. Keller, I. Kay and J. Shmoys, "Determination of the Potential from Scattering Data", Phys. Rev., Vol. 102, No. 2, 557-559, April 15, 1956.

$$x = b^{-2} \tag{16a}$$

$$u = r^{-1} \tag{16b}$$

$$v(u) = 1 - V \tag{16c}$$

$$w = u^2 v^{-1} \tag{16d}$$

In terms of the new variables, equation (14) becomes

$$\frac{\pi - \theta(x)}{2} = \int_o^x \frac{v^{-1/2}(du/dw)}{(x-w)^{1/2}} dw . \tag{17}$$

If we now recognize that the numerator of the integrand is simply an unknown function of w, equation (17) is of Abel type. Denoting the numerator of the integrand in (17) by $g(w)$, we have

$$\begin{aligned} v^{-1/2}(du/dw) \equiv g(w) &= \frac{d}{dw}\left[\frac{1}{2\pi} \int_o^w \frac{\pi - \theta(x)}{(w-x)^{1/2}} dx\right] \\ &= \frac{1}{2} w^{-1/2} - \frac{1}{2\pi} \frac{d}{dw} \int_o^w \frac{\theta(x)dx}{(w-x)^{1/2}} . \end{aligned} \tag{18}$$

From (16d) we see that

$$\frac{du}{dw} = \frac{1}{2} w^{-1/2} v^{1/2} + \frac{1}{2} w^{1/2} v^{-1/2} \frac{dv}{dw} ,$$

and therefore

$$g(w) = \frac{1}{2} w^{-1/2} + \frac{1}{2} w^{1/2} v^{-1} dv/dw . \tag{19}$$

Solving this equation for v in terms of $g(w)$ and noting that $v = 1$ when $w = 0$, we obtain

$$v(w) = \exp \int_{0}^{w} \left[2(w')^{-1/2} g(w') - (w')^{-1} \right] dw' . \qquad (20)$$

Substituting the expression for $g(w)$ from (18) into (20), we obtain

$$v(w) = \exp \left[\frac{-1}{\pi} \int_{0}^{w} \frac{1}{\sqrt{w'}} \frac{d}{dw'} \int_{0}^{w'} \frac{\theta(x)dx}{\sqrt{w'-x}} \right] \qquad (21)$$

Having thus found $v(w)$, we can now calculate $r = u^{-1} = v^{-1/2} w^{-1/2}$ and $V = 1 - v$. The calculation of $V(r)$ is then a parametric one, with w as a parameter.

5. Special Cases of θ vs. b Variation

In principle the procedure of the preceding section can be applied to numerical data for $\theta(b)$. The integral in (21) would have to be evaluated numerically for a large number of values of parameter w; this is clearly a cumbersome procedure. If, instead of evaluating the integral numerically, we could approximate the numerical data for $\theta(b)$ by one of a family of analytic expressions which would permit explicit evaluation of $v(w)$, the procedure would be greatly simplified. We shall now discuss one such family of functions $\theta(b)$. These functions are linear for small value of b, reach a maximum $\theta = \theta_o$ at $b = b_o$ (this implies a soft, or penetrable distribution), and then tend to zero as some inverse power of b. The defining equation for θ is

$$\theta = \theta_o \frac{b}{b_o} \left[\frac{1 + n}{n + (b/b_o)^2} \right]^{\frac{n+1}{2}} \qquad (22)$$

For large b, $\theta \sim b^{-n}$. We are going to consider the special cases $n = 1, 2$. (See Fig. 8.)

For the case $n = 1$ we have

$$\frac{\theta}{\theta_o} = \frac{2b/b_o}{1 + (b/b_o)^2} \tag{23}$$

So that

$$\int_o^w \frac{\theta(x)dx}{\sqrt{w-x}} = \frac{2\pi\,\theta_o}{b_o}\left[1 - \sqrt{\frac{1}{1 + w b_o^2}}\right] \tag{24}$$

and

$$\begin{aligned} v(w) &= \exp\left[-\theta_o b_o^{-2} \int_o^w (w' + b_o^{-2})^{-3/2} (w')^{-1/2} dw'\right] \\ &= \exp\left[-2\theta_o \left[wb_o^2/(wb_o^2+1)\right]^{1/2}\right] \end{aligned} \tag{25}$$

The parametric equations for the normalized plasma density V in this case are then

$$V(w) = 1 - \epsilon^{-2\theta_o(1+1/wb_o^2)^{-1/2}} \tag{26}$$

$$r(w) = w^{-1/2}\,\epsilon^{\theta_o(1+1/wb_o^2)^{-1/2}} \tag{27}$$

It is clear from the above equation that small values of w correspond to large values of r, and conversely. While it is impossible to eliminate w between equations (26) and (27), and thus obtain V as a function of r directly, it is possible to accomplish this approximately for very small or very large r. We are primarily interested in the value of V at $r = 0$, i.e., V_o, and in the manner in which V tends to zero for large r:

$$w \rightarrow \infty,\ r \rightarrow 0,\ V_o = 1 - \epsilon^{-2\theta_o} \tag{28}$$

$$w \longrightarrow 0\,, \quad r \approx w^{-1/2}\,, \quad V \approx 1 - e^{-2\theta_o\sqrt{wb_o^2}} \approx 2\theta_o\sqrt{wb_o^2} \approx 2\theta_o b_o/r \tag{29}$$

For the case $n = 2$ we have

$$\frac{\theta}{\theta_o} = \frac{b}{b_o}\left[\frac{3}{2 + (b/b_o)^2}\right]^{3/2}\,, \tag{30}$$

so that

$$\int_0^w \frac{\theta(x)dx}{\sqrt{w-x}} = \frac{3^{3/2}\theta_o}{2b_o}\left[\frac{\pi}{2} - \sin^{-1}\frac{1-2b_o^2 w}{1+2b_o^2 w} - \frac{2\sqrt{2b_o^2 w}}{1+2b_o^2 w}\right] \tag{31}$$

and

$$\begin{aligned} v(w) &= \exp\left[\frac{-2\cdot 3^{3/2}\theta_o b_o^2}{\pi}\int_0^w (1 + 2w' b_o^2)^{-2}\, dw'\right] \\ &= \exp\left[-\frac{3^{3/2}\theta_o}{\pi(1 + 1/2b_o^2 w)}\right] \end{aligned} \tag{32}$$

The parametric equations for $V(r)$ in this case are:

$$V(w) = 1 - \exp\left[-3^{3/2}(\theta_o/\pi)\,(1 + 1/2b_o^2 w)^{-1}\right] \tag{33}$$

$$r(w) = w^{1/2}\exp\left[\,3^{3/2}(\theta_o/2\pi)\,(1 + 1/2b_o^2 w)^{-1}\right] \tag{34}$$

The behavior of $V(r)$ at $r = 0$ and $r \rightarrow \infty$ is, in this case

$$w \longrightarrow \infty: \quad V_o = 1 - e^{-3^{3/2}\theta_o/\pi} \tag{35}$$

$$w \longrightarrow 0: \quad r \approx w^{-1/2} \qquad V \approx 3^{3/2}(\theta_o/\pi)\cdot 2b_o^2 w \approx \frac{2\cdot 3^{3/2}\theta_o b_o^2}{\pi r^2} \tag{36}$$

The plasma density distribution obtained from these models for two values of maximum deflection angle is plotted in Fig. 9.

We can use the results obtained in this section to study further the relation between maximum deflection angle and maximum normalized plasma density. The results obtained for the two models used in this section are compared with the results for a parabolic distribution in Fig. 10. The relation between the maximum deflection angle and maximum electron density for either of the two models investigated in this section is not very different from that obtained for a quadratic density variation. Given the frequency of the probing wave and the maximum deflection angle, we can again estimate the maximum electron density in the ionized column by the use of eq. (15). If more accuracy, or more information about the distribution is required, we must compare the experimentally observed relation between a scattering angle and impact parameter with those used in this report, pick out one that corresponds more closely than the others, and look up the corresponding electron distribution.

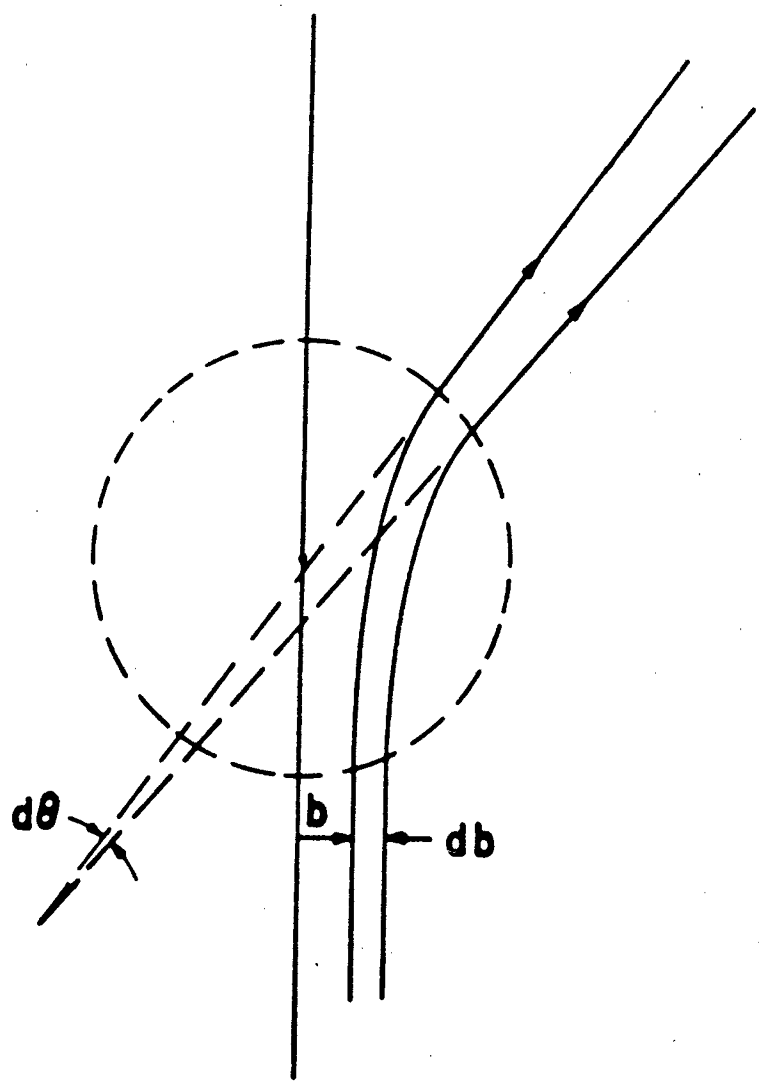

FIG 1a

MRI – 17711

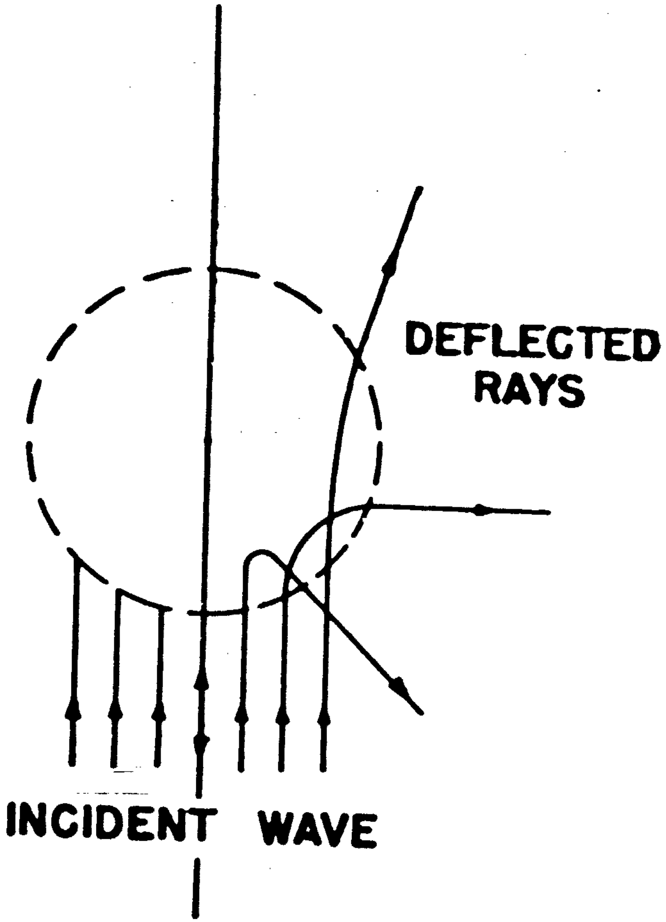

FIG 1b

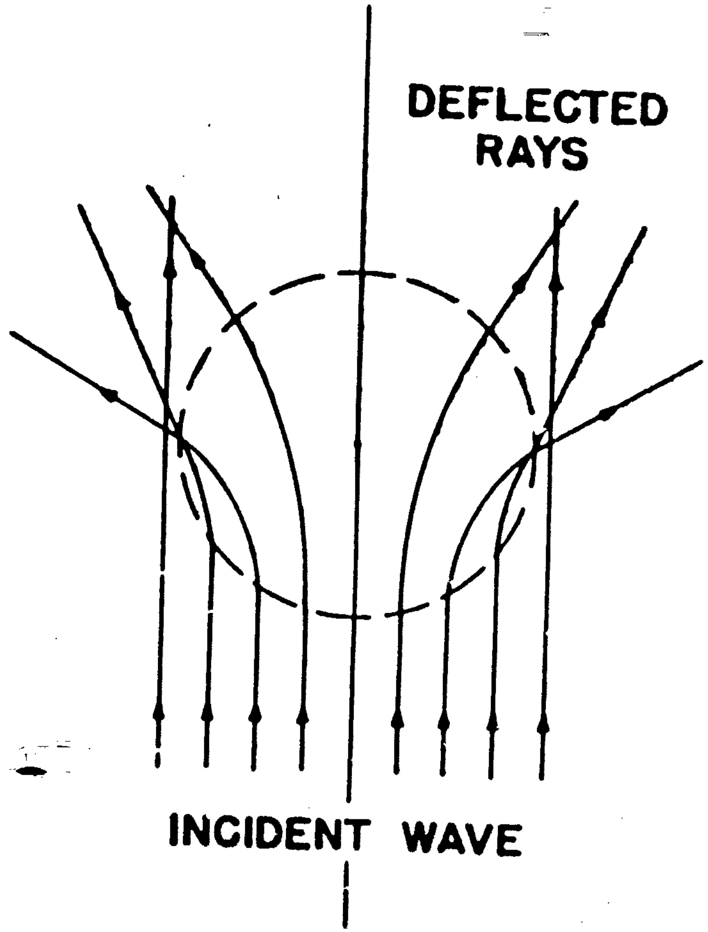

FIG 1c

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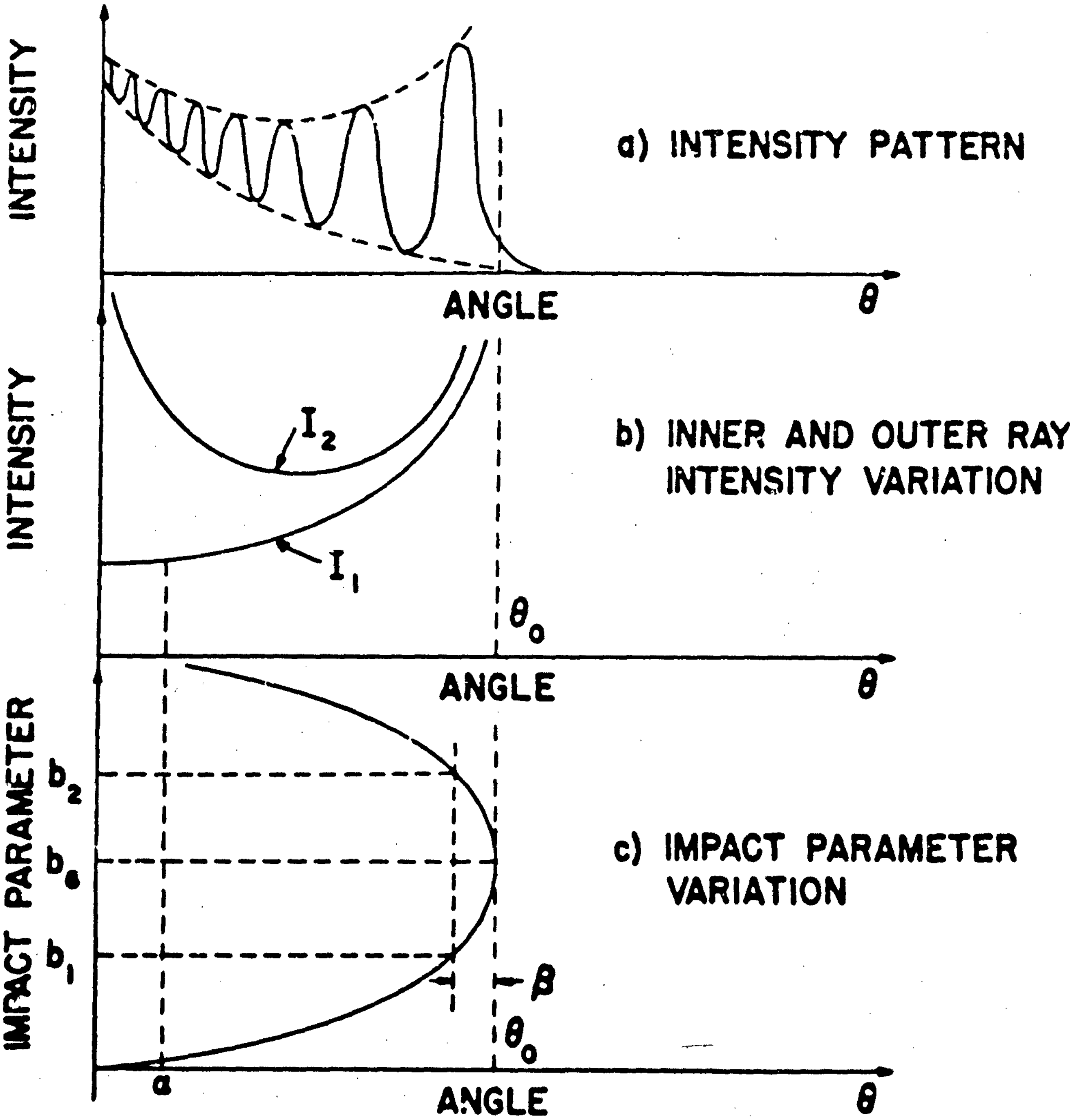

FIG 2
ANGULAR PATTERNS

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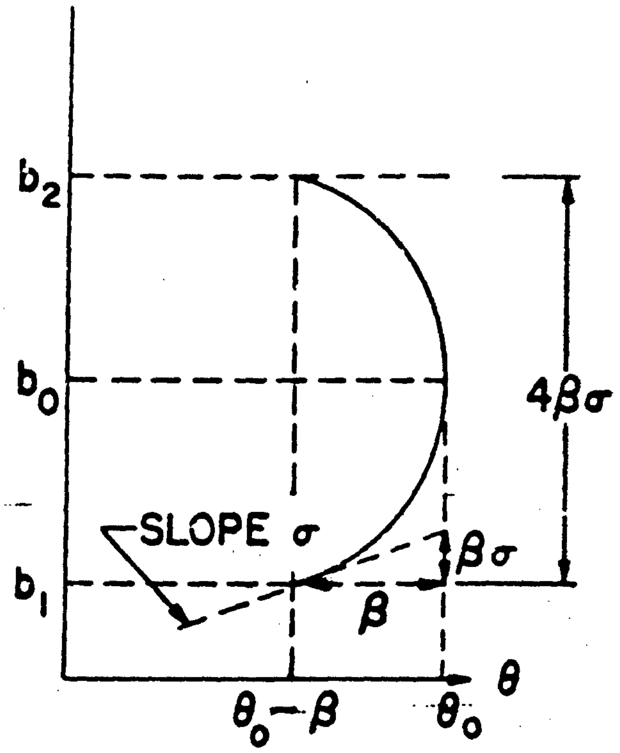

IMPACT PARAMETER VARIATION NEAR THE MAXIMUM

FIG 2d

MRI—17714

FIG 3

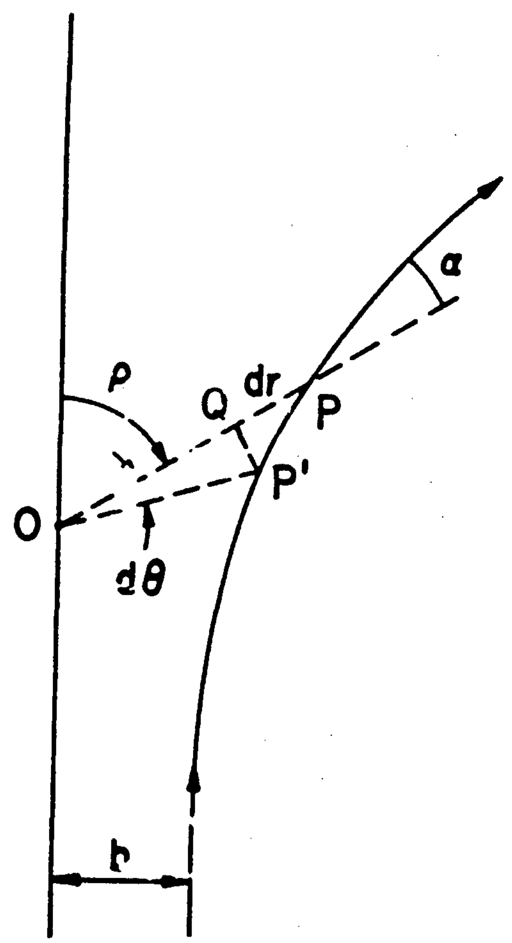

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FIG 4

DEFLECTION ANGLE VS. IMPACT PARAMETER FOR UNIFORM PLASMA CYLINDER

$$V_0 = \frac{\omega_p^2}{\omega^2}$$

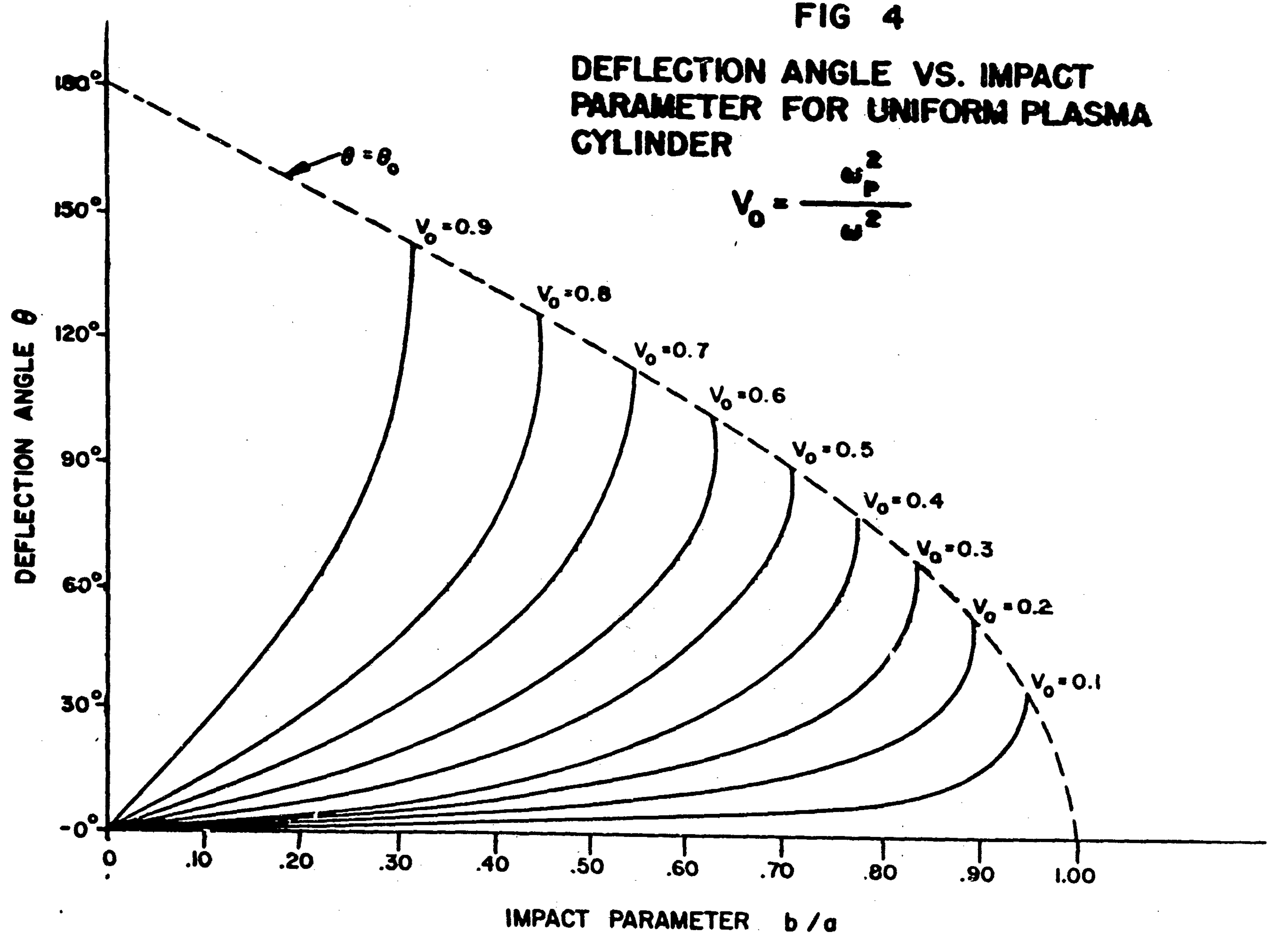

FIG 5
DEFLECTION ANGLE VS. IMPACT PARAMETER FOR A PARABOLIC DISTRIBUTION

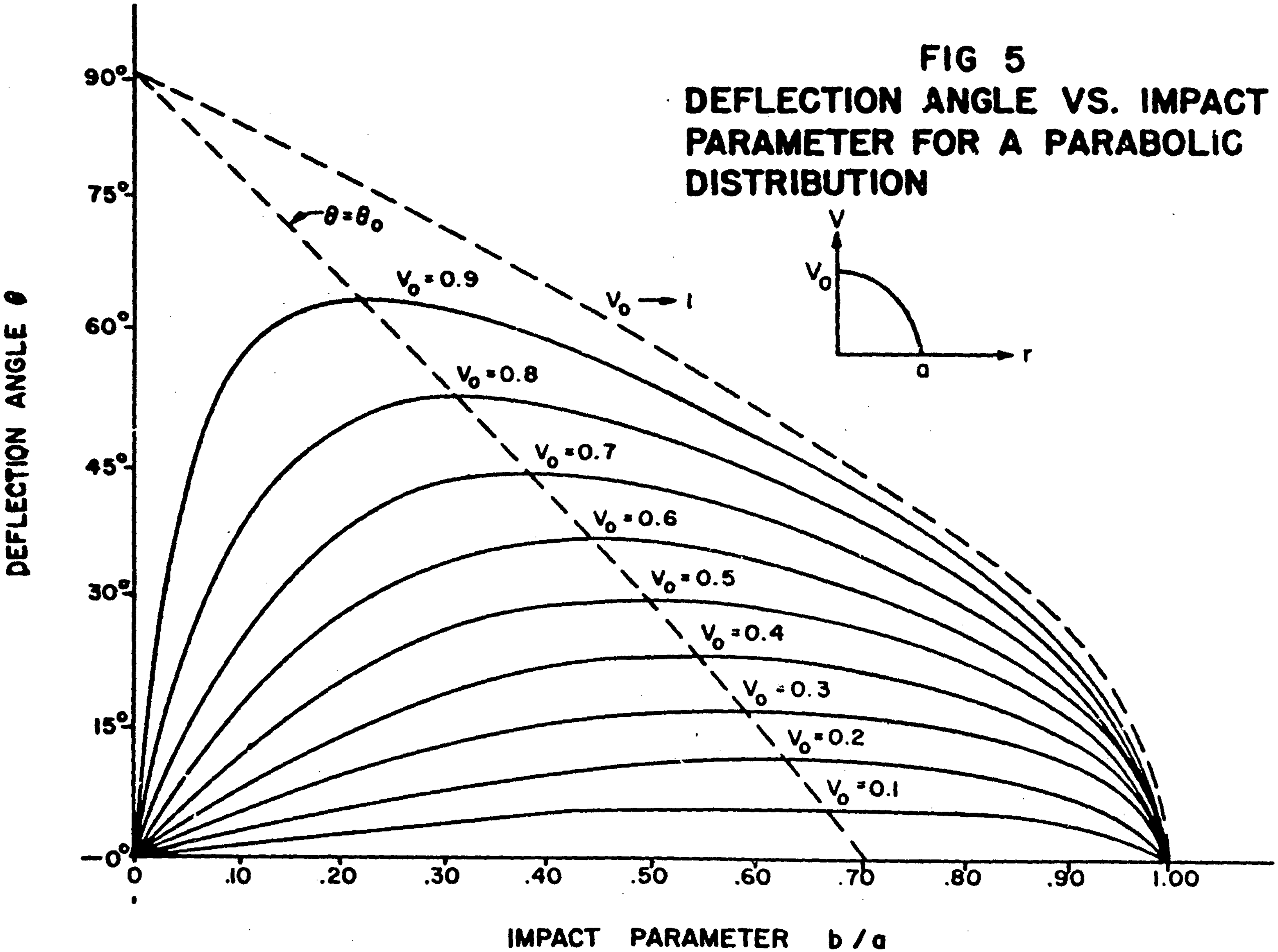

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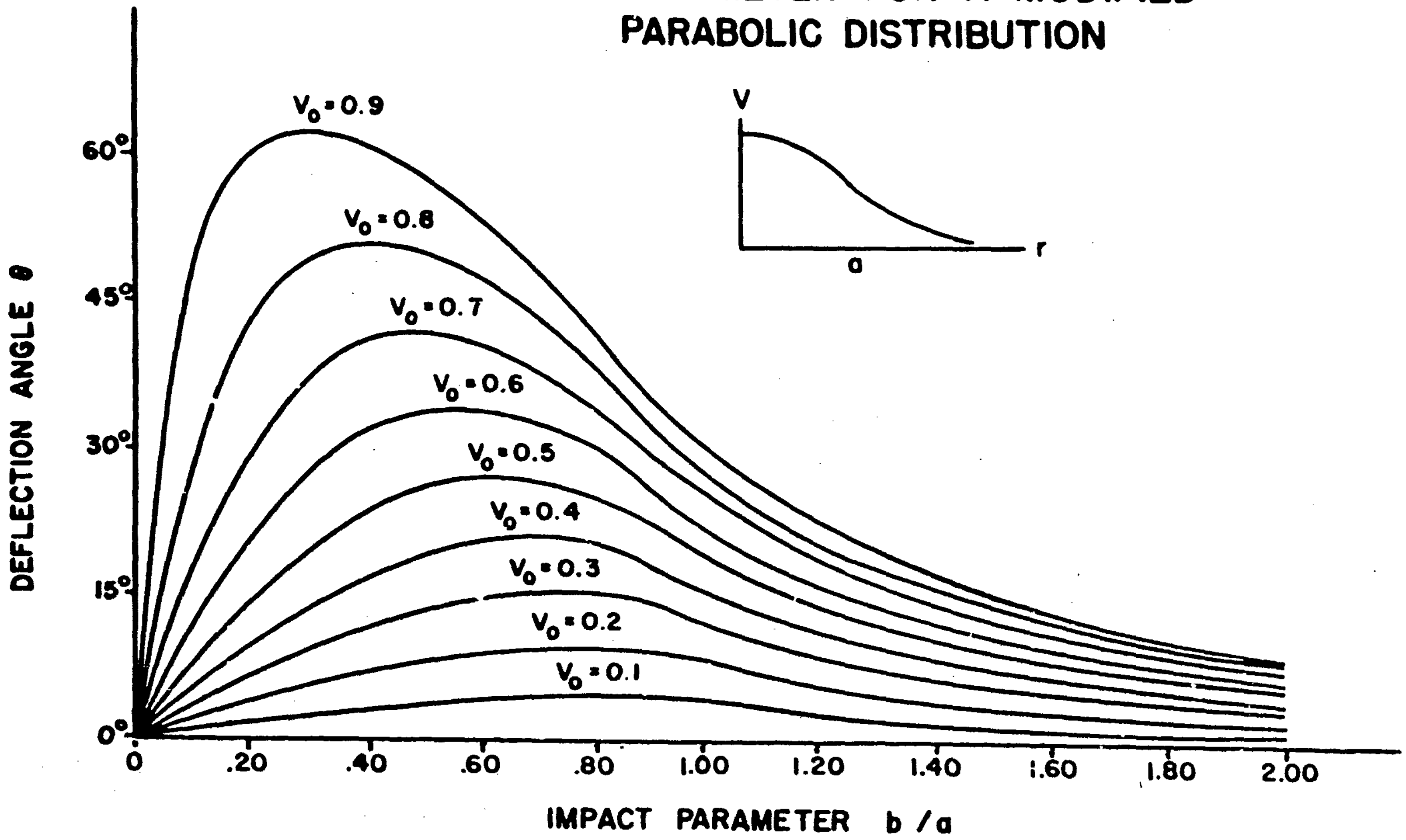
FIG 6

DEFLECTION ANGLE VS. IMPACT PARAMETER FOR A MODIFIED PARABOLIC DISTRIBUTION



MRI-17718

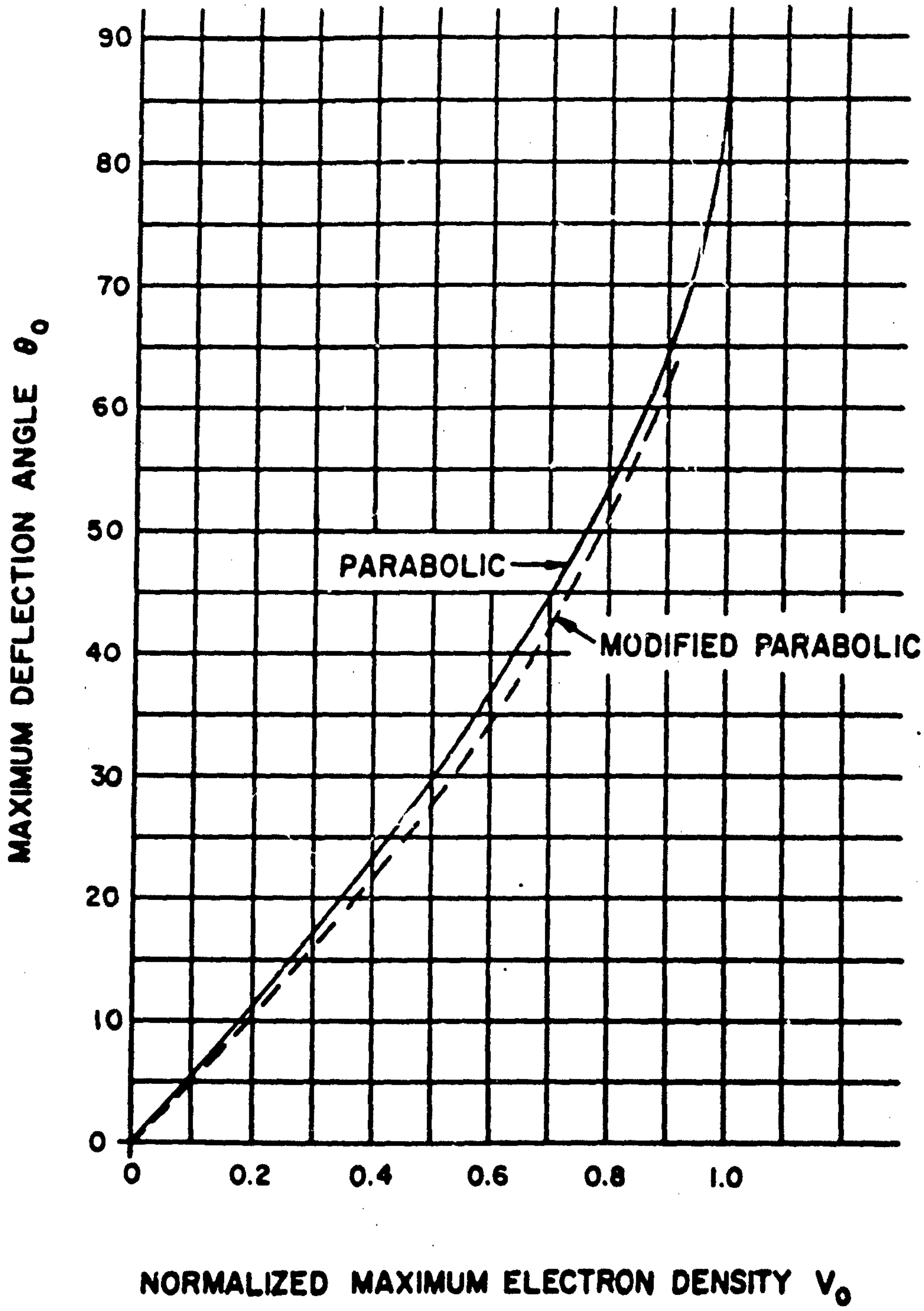
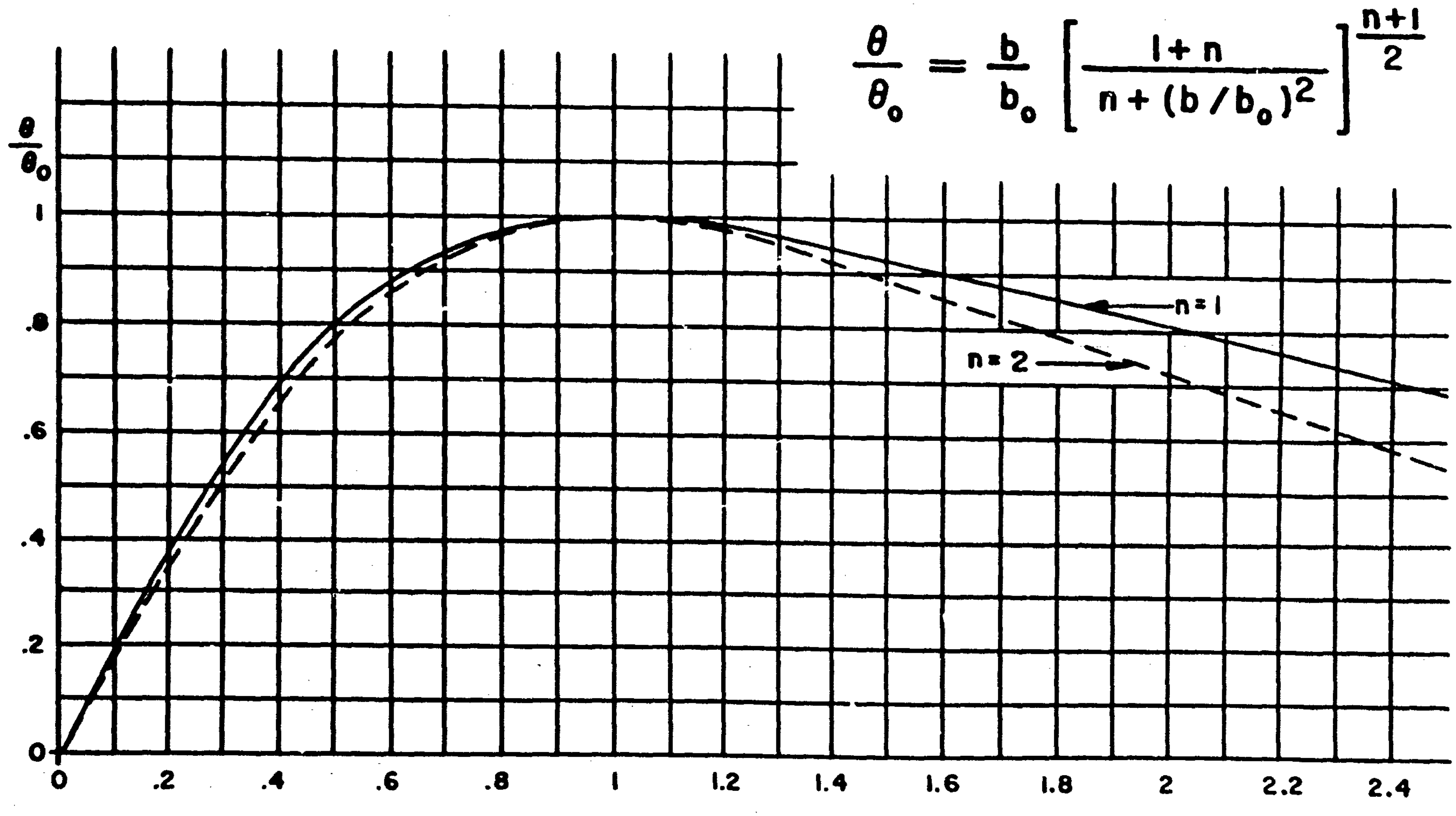


FIG 7

EXTREME RAY DEFLECTION VS. MAXIMUM DENSITY

MRI-17719

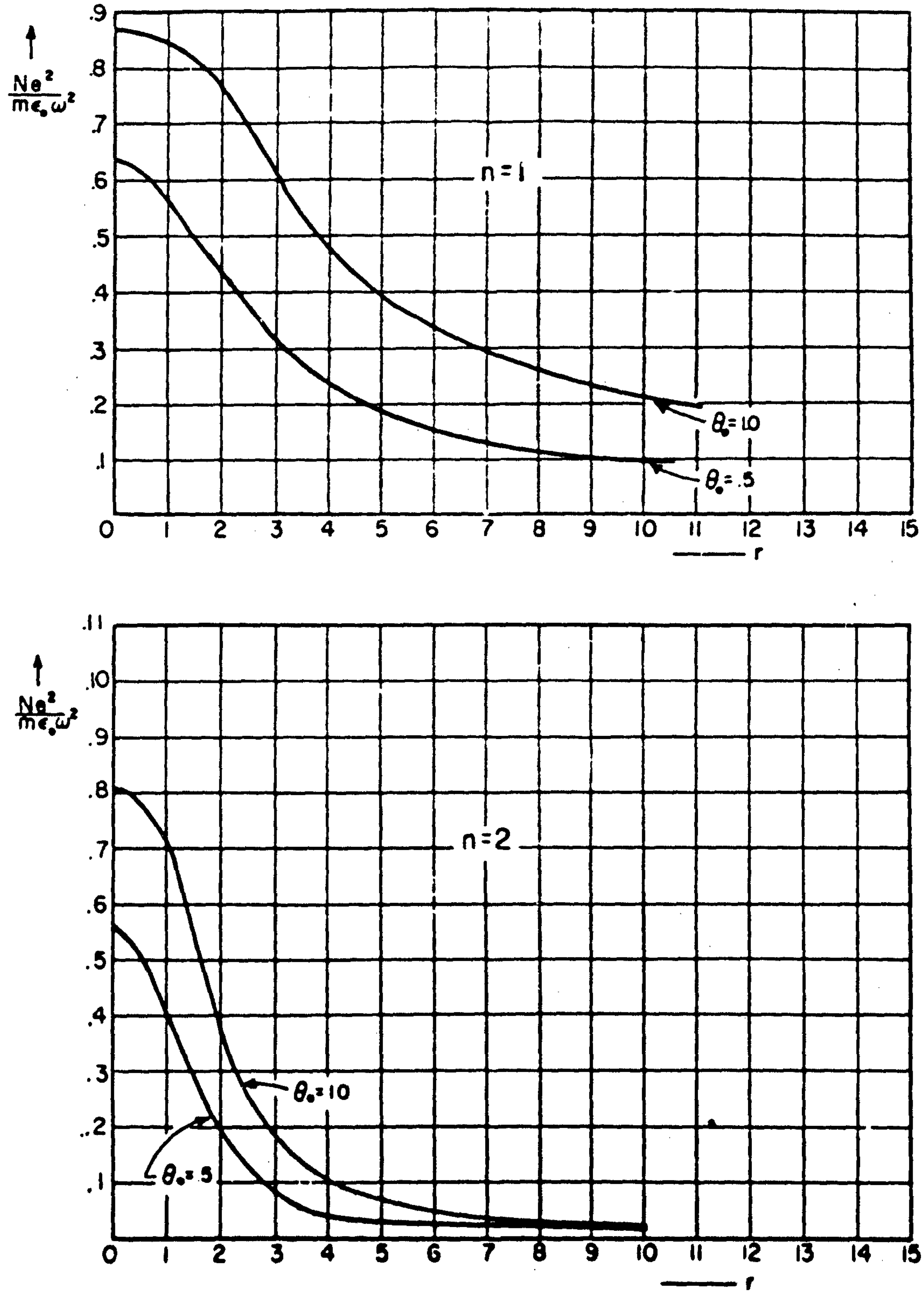
FIG 8
ASSUMED VARIATION OF SCATTERING ANGLE



MRI-17720

FIG 9

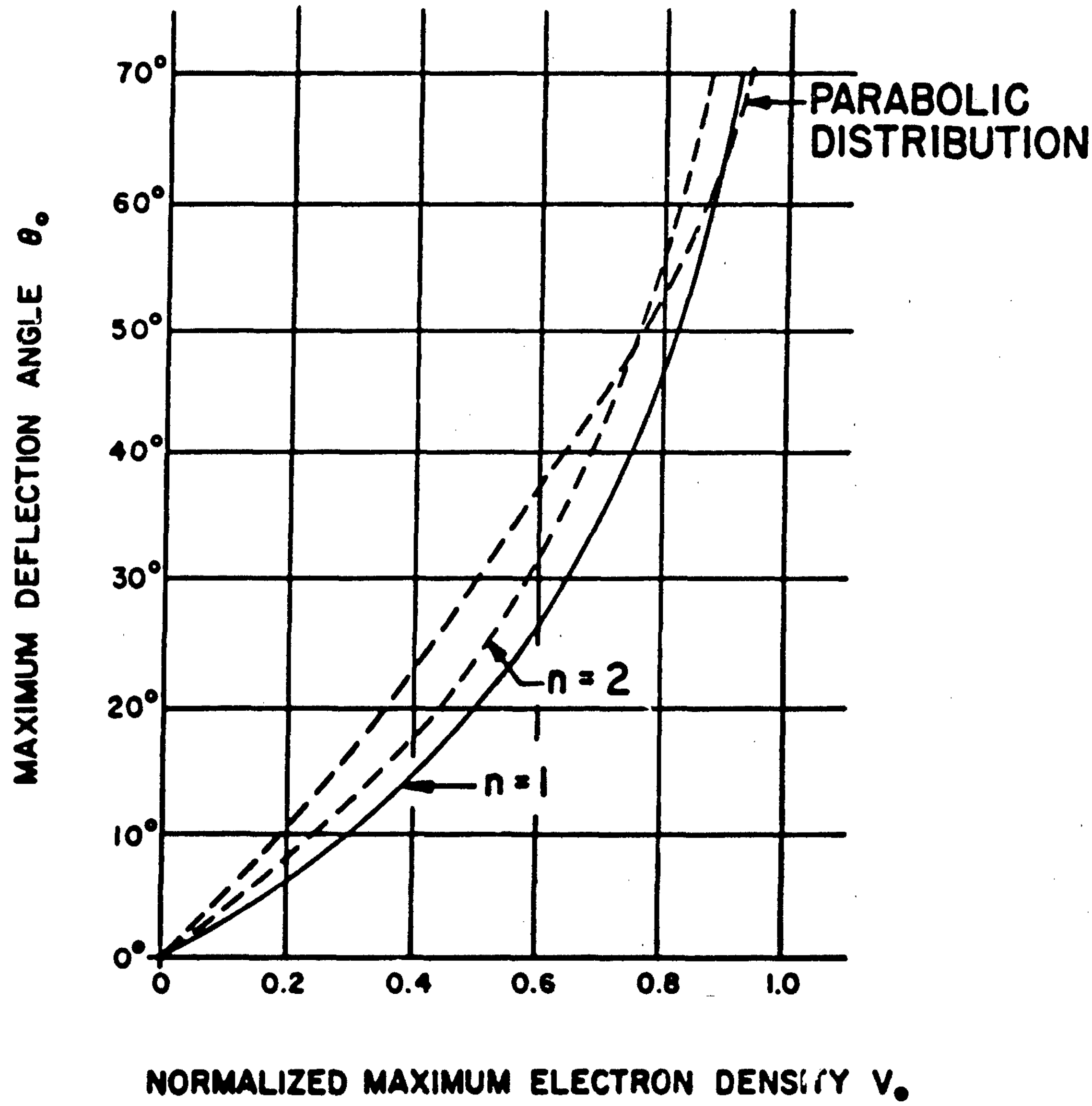
CALCULATED ELECTRON DENSITY DISTRIBUTION



MRI - 17573

FIG 10

EXTREME RAY DEFLECTRON VS. MAXIMUM DENSITY



MRI-17721